Math 250 – Sect.3.4 – Concavity and the Second Derivative

I. CONCAVITY:

Definition of concave up/concave down: *A picture is worth a thousand words!!*

Theorem: Let *f* be a function whose second derivative exists on an open interval (a, b).

1. If _____ for all *x* in (a, b), then *f* is _____ on (a, b).

2. If _____ for all *x* in (a, b), then *f* is _____ on (a, b).

A POINT OF INFLECTION is a <u>point on the curve</u> at which concavity *changes*.

ALSO:

1. *f* is _____ when $f'(x)$ is _____.

2. *f* is _____ when $f'(x)$ is _____.

-example- Consider the function $f(x) = x^3 - 5x^2 - 8x + 5$. Find the intervals on which the function is concave up and concave down, and the locations of any points of inflection.

example- Consider the function $f(x) = \frac{2}{x^2+1}$. Find the intervals on which the function is concave up and concave down, and the locations of any points of inflection.

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*The second derivative ALSO can help determine relative extrema (as did the first derivative, discussed yesterday).

Pictures:

Second Derivative Test: Let f be a function such that $f'(c) = 0$, and the second derivative of f exists in an open interval containing c

1. If $f''(c) < 0$, then f has a relative _____ at $x = c$.

2. If $f''(c) > 0$, then f has a relative _____ at $x = c$.

3. If $f''(c) = 0$, then the test is inconclusive.

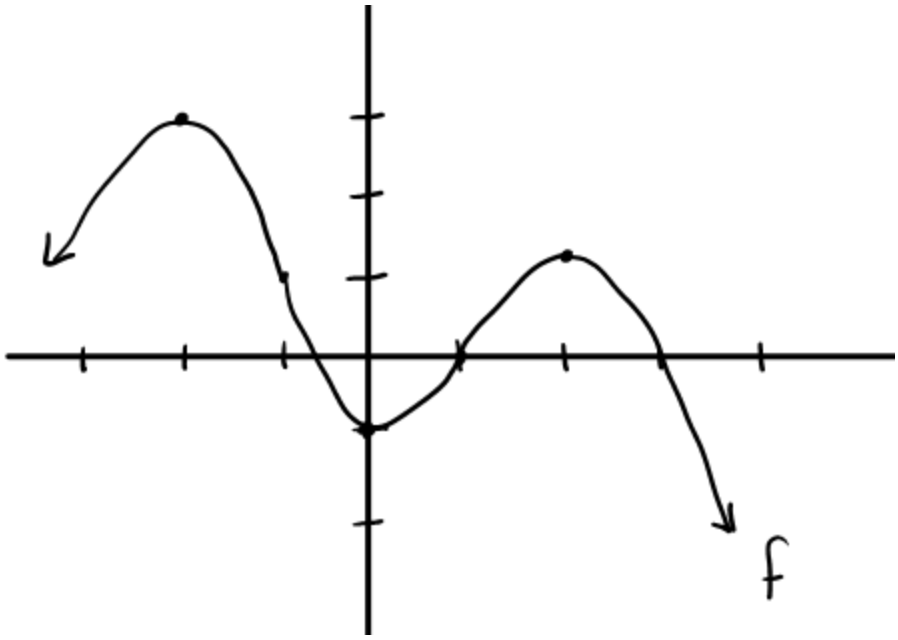
-example- Find all relative extrema for $f(x) = x^3 - 5x^2 + 7x$

Solution 1: First Derivative Test

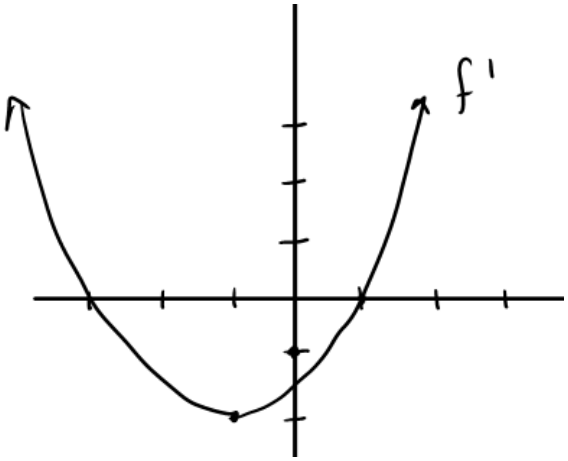
Solution 2: Second Derivative Test

II. Curve Sketching: f, f', f''

-example- Given the graph of f, sketch graphs of f' and f''.



-example- Given the graph of f', answer the given questions about f.



a. On what interval(s) is f increasing?

b. On what interval(s) is f decreasing?

c. For what value(s) of x would f have a maximum?

d. For what value(s) of x would f have a minimum?

e. On what interval(s) is f concave up?

f. On what interval(s) is f concave down?

g. For what value(s) of x would f have a point of inflection?